Technology permeates every aspect of our daily lives, from the sensors that control the traffic signals on our morning commute to the cameras that allow real-time video chats with family around the world. At times, technology may make our lives easier, faster, and more productive. However, does technology do the same in our schools and classrooms? Will the benefits of technology translate to learning reading, mathematics, science, or social studies?

As elementary school mathematics teachers become aware of interactive math simulations, virtual manipulatives, graphics, and dynamic models, we must question ourselves and our motives:

- Is this technology important to my classroom?
- When should my students use technology?
- How will this technology influence student achievement?

Although these questions have been researched ever since the first computers were placed into elementary school classrooms, we do not have solid answers even today.

Researchers have attempted to demonstrate through test scores the advantages of students using a virtual versus concrete base-ten blocks to learn place-value concepts.
particular type of technology, virtual manipulatives. Studies linking technology to student achievement provide a mixed bag of results and conclusions for students’ use of virtual manipulatives in mathematics (Bolyard 2006; Drickey 2000; Kim 1993; Smith 2006; Steen, Brooks, and Lyon 2006; Suh and Moyer 2007; Takahashi 2002; Terry 1995). Questions that link student technology use to student achievement are often difficult to answer. However, teachers at Kennedy Elementary in Alief Independent School District asked a different question. Students are no longer being asked to learn to use technology but to develop skills and learn with technology (ISTE 2007). Following this new research agenda, third-grade math teachers joined a school’s math coach in attempting to discover how students were thinking and interacting mathematically while learning with technology. Using video, the teachers captured interactions of students using virtual base-ten blocks while studying place value. Rather than trying to link technology use and student achievement, the teachers asked, How do students think mathematically when using virtual base-ten blocks to learn place-value concepts?

Examining classroom assessments

In the study, third-grade classes engaged in a unit to learn place value by using base-ten blocks to build and identify quantities and write the corresponding numerals. Third graders are expected to read, write, and describe numbers through 999,999 and compare numbers through 9,999. To compare how students interacted with base-ten blocks during the place-value unit, the team had students in two classes
use concrete base-ten blocks while students in two other classes used virtual base-ten blocks. Teachers in all four classes followed the same lesson plans, enVisions curriculum, and investigations textbook series. Students worked in pairs in all four classrooms. However, students who worked in pairs at the computer switched at ten-minute intervals to give each student an equal opportunity to manipulate the mouse.

During the study, the math coach randomly chose one pair of students to be videotaped each day so that the team could observe students’ interactions with the virtual blocks, hear students’ conversations (including counting), and describe their construction of quantities (Burris 2010). Similarly, the coach randomly videotaped one pair of students using concrete base-ten blocks. Video recordings provided the opportunity for repeated viewings of student thinking and interactions with both types of blocks (Knoblauch 2009). Field notes and student work were also collected to help describe the interactions.

**Virtual and concrete base-ten blocks**

The virtual base-ten blocks used in the study were designed by enVision as part of their e-Tools software. The blocks can be described as *groupable models* because the blocks can be grouped and regrouped into units, tens, hundreds, and thousands (Van de Walle and Lovin 2006). Additionally, a numeral counter was available. As students placed blocks onto the screen, the counter recorded the results. These dynamic virtual models allowed students to interact with and manipulate the blocks using different tools. For instance, students could use the hammer tool to break quantities apart and the glue tool to regroup quantities (see fig. 1).

Students who used the concrete models worked with Interlox™ base-ten blocks (see fig. 2). These blocks are also considered groupable because students can build representations of one ten, one hundred, or one thousand. The blocks were chosen because they can be manipulated similarly to the virtual blocks.
Place-value learning experiences

Working in pairs, students engaged with the virtual base-ten blocks in a computer lab or with the concrete blocks in the classroom each day. To align with Dienes’s (1969) dynamic principle of unstructured play with manipulatives, the teachers began the unit with time for students to explore the blocks and become familiar with the tools (see table 1).

How did students interact?

Early elementary mathematics education focuses on the three components of number: the written numeral, the quantity, and the verbal or spoken number (Wright et al. 2002). While the teachers observed student interactions, they used a video protocol that accounted for these components (see fig. 3). They wanted to watch and record how students used the concrete and virtual blocks to build a quantity, count the quantity, and write the numeral. The video recording sheet also included research about place value (Fuson et al. 1997b; Fuson et al. 1997a) and the UDSSI (unitary, decade, sequence, separate, integrated) model (see table 2).

The purpose of including the model was to help clarify the sequence of place-value learning and identify those characteristics of students. The model provided a lens to observe the students’ conceptual structures of place value while they interacted with the manipulatives. After the conclusion of the unit, the teachers used the video protocol as they watched students’ interactions.

Components of number

Using both concrete manipulatives and virtual manipulatives, students built numbers by starting with the largest digit, whether it was

<table>
<thead>
<tr>
<th>Instructional timeline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>Exploration with virtual or concrete blocks</td>
</tr>
<tr>
<td>Day 2</td>
<td>Building hundreds</td>
</tr>
<tr>
<td>Day 3</td>
<td>Making numbers with hundreds, tens, and units</td>
</tr>
<tr>
<td>Day 4</td>
<td>163 stickers: Noncanonical numbers</td>
</tr>
<tr>
<td>Day 5</td>
<td>Build and write: Expanded notation</td>
</tr>
<tr>
<td>Day 6</td>
<td>Thousands</td>
</tr>
<tr>
<td>Day 7</td>
<td>Thousands</td>
</tr>
<tr>
<td>Day 8</td>
<td>Greater numbers</td>
</tr>
<tr>
<td>Day 9</td>
<td>Comparing numbers</td>
</tr>
<tr>
<td>Day 10</td>
<td>Posttest: Place value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conceptual structure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unitary</strong></td>
<td>Students use a count-by-ones strategy and can identify “a whole word to a whole quantity.” They cannot partition 32 into 3 tens and 2 units.</td>
</tr>
<tr>
<td><strong>Decade</strong></td>
<td>Students use a count-by-ones strategy and may start to relate the number words to separate quantities. For example, when counting 32, a student relates “thirty” to thirty objects and “two” to two objects.</td>
</tr>
<tr>
<td><strong>Sequence</strong></td>
<td>Students understand tens and units; a student may count, “Ten, twenty, thirty, thirty-one, thirty-two.”</td>
</tr>
<tr>
<td><strong>Separate</strong></td>
<td>Students understand that digits separated are tens and units. Students may count groups as “1 ten, 2 tens, 3 tens, and 1, 2 units.”</td>
</tr>
<tr>
<td><strong>Integrated</strong></td>
<td>Students can move fluidly between sequence and separate conceptualizations. Students can identify the 3 in the 32 as both thirty and as 3 tens.</td>
</tr>
</tbody>
</table>
This example of the construction of quantity with concrete models depicts the building and similar recording of the numeral 873.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expanded Form</th>
<th>Word Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 hundreds, 7 tens, 3 units</td>
<td>800 + 70 + 3</td>
<td>8 hundred, 7 tens, 3 ones</td>
</tr>
</tbody>
</table>

During the construction of quantities, students in both groups favored one counting strategy. Of 138 representations of number analyzed by video, 128 constructions were observed using a count-by-tens and a count-by-groups strategy, more specifically, the integrated conception (Fuson et al. 1997b). Students in both groups demonstrated what Fuson (1998) calls "the place-value meaning of number words." The counting showed students’ ability to count by tens or count by groups. These two strategies demonstrated a conceptualization of single groups of ten (Fuson 1998). Different types of number word sequences directly align to the conceptualization of place value within the UDSSI model (see Table 2).

Why is this important? Students in both groups used virtual and concrete base-ten blocks in similar ways: They built quantities from left to right; they could count the quantity and could write the corresponding numeral and expanded form similarly. Regarding the components of number, they
interacted with virtual base-ten blocks in the same way as they did with the concrete ones.

**Place value (UDSSI)**

The instructional team incorporated the UDSSI model into the study to frame students' thinking and abilities regarding place value. The model provided the teachers with a conceptual model for the learning and understanding of place value (Fuson et al. 1997a; Fuson et al. 1997b). They attempted to delineate the conceptual stages for students by listening to the method of counting, watching the construction of numbers, and observing the representations of numerals.

Video analysis suggests that students in both groups were able to correctly describe the quantities as tens and ones and make the connection to the written symbol; this is described as the integrated structure (Fuson et al. 1997b). The integrated conception of the UDSSI model describes the ability of students to move seamlessly between the count-by-tens and the count-by-groups structures. For example, students may be able to link seventy-two and seven tens and two ones with the numeral 72. The integrated conception suggests that students can move between the two structures. The connection is made with the written numeral and the quantity (Fuson 1998).

While constructing 6256 with virtual manipulatives, Portia and Jacquelin talked through their work as they clicked on the virtual blocks.

**Figure 6**

Portia and Jacquelin used a box with an X to represent the thousand cube, moving fluidly by identifying the numeral and the component places of 6256.

<table>
<thead>
<tr>
<th>Place Value Chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

**Table 3**

Using virtual and concrete manipulatives, students constructed numbers within the UDSSI structures.

<table>
<thead>
<tr>
<th></th>
<th>Unitary</th>
<th>Decade</th>
<th>Sequence</th>
<th>Separate</th>
<th>Integrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>55</td>
</tr>
<tr>
<td>Virtual</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>73</td>
</tr>
</tbody>
</table>

"We are building 6256." Portia stated.

As Jacquelin clicked on the quantities, beginning with the thousands and moving to the units, she counted, “One, two, three, four, five, six thousands; two hundreds; one, two, three, four tens; and six ones.”

Portia saw a discrepancy: “Wait, you need one more ten; you only have four.”

Using the arrange tool, Jacquelin organized the blocks while Portia recorded the solution (see fig. 6). Note that Jacquelin used a box with an X to represent the thousand cube. In this respect, Portia and Jacquelin moved fluidly by identifying the numeral and the component places, thereby demonstrating the integrated conception of the UDSSI model (see table 3).

Regardless of which manipulative they used, most students operated within the integrated.
conceptualization of place value. Students could count by tens and count by groups. The virtual models offered the same support and interaction as the concrete base-ten blocks.

**Renaming numbers**

As a precursor to computation, building nonstandard forms, or noncanonical representations, of number can assist students in the renaming of numbers (Van de Walle and Lovin 2006). For example, a student using the algorithm to solve $62 - 27$ would rename $62$ as $5$ tens and $12$ ones to subtract. During the two lessons for renaming numbers, students in both groups generated a total of twenty-five nonstandard numbers. Using concrete manipulatives, students constructed ten representations, and students using virtual manipulatives created fifteen nonstandard forms. Although the students using the virtual manipulatives generated more nonstandard representations, the information supplied by the videotaped sessions and student work offered a clearer picture of the differences in each groups’ thinking about place value.

One dissimilarity that teachers noted between the groups was the ease of construction of the nonstandard numbers for those using the virtual manipulatives. A distinct difference was that students using the virtual manipulatives “reused” the quantity on the screen. They used the hammer and glue tools to show various representations without clearing the screen or starting over. However, students using the concrete models had to “trade” the blocks to construct the quantity.

When Kim and Nuri were finding nonstandard forms of 163, Kim built 163 by first adding $1$ hundred followed by $6$ tens and $3$ ones to display the standard form. Using the hammer tool to break apart the blocks, Kim broke the hundreds into $10$ tens.

"Is that still $163$?" asked Nuri.

“Yes, look: It’s over here." Kim pointed to the counter in the lower portion of the screen.

Teachers noted that the pair had used the numeral counter to help them build quantities of 163. Next, the girls created $16$ tens and $3$ ones. Nuri counted the tens as “ten, twenty, thirty, forty,” through 160. The pair continued with $1$ hundred and $63$ ones on the board, using the hammer tool to experiment. They broke the hundred flat into $10$ tens.

"We could do that, $10$ tens and $63$ ones." Kim suggested. By using tools available with the virtual manipulatives—including the hammer tool and the place-value chart—Kim and Nuri found seven nonstandard representations for 163, including $14$ tens and $23$ units (see fig. 7).

Within the virtual manipulative group, the standard form of the number was constructed and students used available virtual tools, including the counter, hammer, glue, and place-value tools, to find solutions. Within the concrete manipulative group, students used counting strategies and benchmark numbers to generate the nonstandard form. The counting strategies and benchmarks may have benefited students with regard to the construction of some nonstandard representations. However, the strategy may have limited the number of ways that students could rename numbers, depending on students’ facility with the forward
number word sequence and visualization of the model. The virtual tools allowed students to realize the potential for renaming numbers without those restrictions.

**What are the benefits of virtual base-ten blocks?**
This study suggests that students interacting with virtual or concrete base-ten blocks are capable of mathematical thinking of place value. Because students use and interact with both manipulatives in similar ways, evidence supports the use of virtual or concrete manipulatives. Regarding place value, the study suggests that students construct quantities, write numerals, and count or identify quantities similarly with concrete or virtual manipulatives.

Virtual manipulatives provided support for learning nonstandard representations or renaming numbers. These nonstandard forms are distinctly interconnected with multidigit addition and subtraction, specifically the multidigit algorithm (Hiebert and Wearne 1996). The available virtual tools, including the hammer and glue tools to break apart and reorganize quantities, allowed students to construct multiple nonstandard forms.

Using virtual blocks, students could accurately build quantities, could write numerals, and could count quantities related to place-value concepts. However, students could also build nonstandard forms or could rename numbers with the tools provided by the virtual models. Students who used virtual blocks could compose and decompose numbers more readily than those who used concrete blocks. These virtual tools are a viable manipulative for students to use when constructing knowledge of place-value concepts. This study suggests that by interacting with virtual manipulatives, students can demonstrate mathematical thinking of place-value concepts with the added benefit of constructing nonstandard representations that are directly linked to the multidigit algorithm.

**Conclusions**
Before elementary school mathematics teachers adopt a technology—and more important, before students interact with technology—we must ask, What is the purpose of the technology? As students at Kennedy Elementary School learned place-value concepts with virtual models, the purpose of the virtual base-ten blocks became clear. Students used the virtual blocks to construct and count quantities and to write and identify numerals just as they would with concrete models. Surprisingly, the virtual models benefited students in renaming numbers.

After reviewing the videos, team members were amazed that students used the concrete and virtual blocks in a similar way. Students’ interactions with virtual base-ten blocks in this study were similar to students’ interactions with concrete blocks. The virtual models were advantageous to students as they generated nonstandard numbers more efficiently using technology.

When thinking about using virtual manipulatives in your classroom, do not ask whether virtual models are “concrete” but rather how students will interact with the models and how they will think mathematically when using them. As Clements (1999) suggested, children’s interactions with manipulatives should be the emphasis, not the manipulatives themselves. Before teachers adopt a technology in elementary school math classrooms, they must ask themselves (1) What is the purpose of the technology or virtual manipulative, and (2) how will students interact with and think mathematically when using the technology? Kennedy Elementary School students interacted with virtual manipulatives to think mathematically about place-value concepts.

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Justin T. Burris, justin.burris@gmail.com, is a math coach at Kennedy Elementary School in the Alief Independent School District in Houston, Texas. He is a visiting assistant professor of Mathematics Education at the University of Houston and is interested in students’ mathematical interactions with virtual models and manipulatives.
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